

# Entanglement of arbitrary spin modes in an expanding universe

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Pair particle creation is a well-known effect on the domain of field theory in curved space-time. It is shown that the entanglement generations for spin-0 and spin-1/2 modes are different in Friedmann-Robertson-Walker (FRW) space-time. We consider the spin-1 particles in FRW space-time using Duffin-Kemmer-Petiaio (DKP) equation and obtain a measure of the generated entanglement. Also, we consider the spin-3/2 particles. We argue that the absolute value of the spin does not play any role in entanglement generation and the differences are due to the bosonic or fermionic properties.

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## I. INTRODUCTION

Because of the importance and the fundamental role of entanglement in quantum information processing and quantum computing, a lot of researches have been done on entanglement generation, variation and degradation in various domains. In non-relativistic limits, entanglement has been extensively studied. Recently, relativistic quantum information processing has attracted a lot of interests [1–4]. The world is fundamentally relativistic, therefore, understanding entanglement in space-time is ultimately important. It is realized that relativity plays a significant role in quantum entanglement and related quantum protocols, such as quantum teleportation. This point is justified by quantum optics, which is well established on the basis of not only quantum theory but also special relativity in nature. Most of EPR-type experiments have been performed by photon pairs. In addition, experiments of quantum teleportation have been extensively carried out by photons [5–9].

There are some qualitative differences between non-relativistic entanglement and relativistic one. It has been shown that entanglement is an observer dependent property in non-inertial frames. I. Fuentes *et. al.* have showed that the entanglement of both bosonic and fermionic modes degrade by acceleration [10, 11]. In addition, quantum discord and entanglement of pseudo-entangled spinor modes in non-inertial frames have been studied [12].

Recently, scalar and spinor field modes have been investigated in an expanding universe. It has been shown

that in both cases, the expanding universe generates entanglement between field modes [13].

A separable vacuum state in distant past time appears as an entangled state in far future time because of the expanding universe. However, there are differences between the entanglement of scalar boson fields and spinor fermion fields [14, 15].

It has been shown that the entanglement of massive boson modes is a monotonically decreasing function with respect to the momentum of the modes while for the fermion modes, there is an optimum point for the momentum of the modes. For the spin-1/2 case, there is no entanglement between the zero momentum modes while for the spin-0 case the maximum entanglement is related to the zero momentum modes. As a common behavior between the spin-0 and spin-1/2 modes, it has been shown that there is no entanglement for massless bosons or fermions [15].

To understand the origin of the differences between the generated entanglement in an expanding universe for spin-0 and spin-1/2 modes, we investigate the higher spin modes. Firstly, we consider the spin-1 modes using Friedmann-Robertson-Walker (DKP) equation in Duffin-Kemmer-Petiaio (DKP) space-time and work out a measure for the generated entanglement. The same procedure will be applied on spin-3/2 modes using Rarita-Schwinger equation.

This paper is organized as follows: In section II, we consider the DKP equation in FRW expanding universe specially in two dimensions. In section III, we calculate the entanglement entropies of spin-1 and spin-0 particles and compare them to each other. Also, we investigate the variation of entanglement with respect to the parameters of expansion, and the momentum of any the mode. In section IV, we consider the spin-3/2 particles using

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Rarita-Schwinger equation and work out the generated entanglement and compare it with the entanglement of spin-1/2 modes. Conclusions are presented in section V.

## II. DKP EQUATION IN FRIEDMANN-ROBERTSON-WALKER SPACE-TIME

It is well-known that Klein-Gordon and Dirac equation describe particles with spin-0 and spin-1/2 in flat Minkowski space-time, respectively. Scalar and spinor fields have been considered in more details in curved space-time. There are various ways of formulating a relativistic wave equation describing the dynamical states of a massive vector boson, such as Proca equation, Duffin-Kemmer-Petiau equation and Weinberg-Shay-Good equation [16–19]. We employ DKP equation for considering vector bosons in curved space-time. Before starting the study of the DKP in curved space-time, we notice that it is similar to the Dirac equation in Minkowski space-time as follows:

$$(i\beta^\mu \partial_\mu - m)\Psi = 0, \quad (1)$$

where, the  $\beta^\mu$ -matrices are generalization of the Dirac gamma matrices, satisfying an algebra ring, which for spin-1 is

$$\beta^\lambda \beta^\mu \beta^\nu + \beta^\nu \beta^\mu \beta^\lambda = \eta^{\lambda\mu} \beta^\nu + \eta^{\mu\nu} \beta^\lambda. \quad (2)$$

Generally,  $\beta^\mu$ 's represent four  $16 \times 16$  reducible matrices, which decompose into three separate representations, a one dimensional trivial, a five dimensional spin-0 and a ten dimensional spin-1 representations [20–23]. For the the 10-dimensional spin-1 representation,  $\beta^\mu$  matrices given by

$$\beta^0 = \begin{pmatrix} 0 & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 3} & \mathbf{1}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 1} & \mathbf{1}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{pmatrix}, \quad (3)$$

$$\beta^i = \begin{pmatrix} 0 & \mathbf{0}_{1 \times 3} & \mathbf{K}^j & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -i\mathbf{S}^j \\ -\mathbf{K}^{j\dagger} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 1} & -i\mathbf{S}^j & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{pmatrix}, \quad (4)$$

where,  $\mathbf{S}^i$ 's are the standard  $(3 \times 3)$  spin-1 matrices and  $\mathbf{K}^i$ 's denote  $(1 \times 3)$  matrices with elements  $\mathbf{K}_j^i = \delta_j^i$ .

In curved space-time, we can use the tetrad formalism to obtain the generalized DKP equation

$$\left( i\tilde{\beta}^\mu (\partial_\mu + \frac{1}{2}\omega_{\mu ab}\mathcal{S}^{ab}) - m \right) \Psi = 0, \quad (5)$$

where,  $\mathcal{S}^{ab} = [\beta^a, \beta^b]$  and  $\tilde{\beta}^\mu$ 's are the Kemmer matrices in curved space-time and they are related to Minkowski space-time  $\beta^\mu = e_a^\mu \beta^a$  with the following tetrad relation

$$e_a^\mu e_b^\nu \eta^{ab} = g^{\mu\nu}, \quad e_a^\mu e_{b\mu} = \eta_{ab}, \quad e_a^\mu e_\mu^b = \delta_a^b. \quad (6)$$

Also, the spin connections  $\omega_{\mu ab}$  are given by

$$\omega_{\mu ab} = e_a^l e_b^j \Gamma_{j\mu}^l - e_b^j \partial_\mu e_{aj}, \quad (7)$$

where,  $\Gamma_{j\mu}^l$ 's are the affine connections, which are obtained by the space-time metric elements. Specifically, we consider a two dimensional FRW expanding space-time with line element

$$ds^2 = C^2(\eta)(d\eta^2 - dx^2), \quad (8)$$

where,  $\eta$  is the conformal time, and the conformal scale factor,  $C$ , is given by

$$C(\eta) = (1 + \epsilon(1 + \tanh \rho\eta))^{1/2}, \quad (9)$$

with positive real parameters  $\epsilon$  and  $\rho$ , controlling the total volume and rapidity of the expansion. Primarily, the entanglement between the modes of a quantum field in a curved space-time can be investigated in special states where the space-time has at least two asymptotically flat regions. According to Eq. (9), in the distant past and far future, the space-time becomes Minkowskian, since  $C(\eta)$  tends to  $1 + 2\epsilon$  for  $\eta \rightarrow +\infty$  and it tends to 1 for  $\eta \rightarrow -\infty$ . In the intermediate region, the concept of the particle breaks down.

For simplicity, we restrict ourselves to solve the  $(1 + 1)$ -dimensional DKP equation in FRW space-time. The DKP equation can be obtained in the following form [24]

$$\left[ \beta^0 \partial_\eta + ik\beta^1 - \frac{\dot{C}}{C}(\beta^1)^2 \beta^0 + iCm \right] \tilde{\Psi} = 0, \quad (10)$$

where,  $\Psi(\eta, x) = e^{i\vec{k} \cdot \vec{x}} \tilde{\Psi}(\eta)$  and  $\tilde{\Psi}(\eta)$  has ten components as follows

$$\tilde{\Psi}(\eta) = \begin{pmatrix} \varphi \\ P \\ Q \\ R \end{pmatrix}, \quad (11)$$

where,  $P, Q$  and  $R$  are  $3 \times 1$  vectors and  $\varphi$  denotes a scalar. The Kemmer matrices representation for a suitable arrangement of the components of these vectors and  $\varphi$  leads to the following equations

$$\begin{aligned} iCm\Phi &= -(\partial_\eta + \frac{\dot{C}}{C})\mathcal{X} + ik\Theta, \\ iCm\mathcal{X} &= -\partial_\eta\Phi, \\ iCm\Theta &= -ik\Phi. \end{aligned} \quad (12)$$

Where

$$\Phi = \begin{pmatrix} P_2 \\ P_3 \\ Q_1 \end{pmatrix}, \quad \mathcal{X} = \begin{pmatrix} Q_2 \\ Q_3 \\ P_1 \end{pmatrix}, \quad \Theta = \begin{pmatrix} R_3 \\ -R_2 \\ \varphi \end{pmatrix}. \quad (13)$$

The third component of  $R$  is vanished. We can obtain an independent equation for  $\Phi(\eta)$  as follows

$$\left( \frac{d^2}{d\eta^2} + k^2 + C^2 m^2 \right) \Phi(\eta) = 0, \quad (14)$$

and  $\mathcal{X}(\eta)$  and  $\Theta(\eta)$  satisfy the following equations

$$\mathcal{X} = \frac{i}{Cm} \partial_\eta \Phi, \quad \Theta = -\frac{k}{Cm} \Phi. \quad (15)$$

According to the FRW matrices and using Eq. (14) we

$$\Phi_{in}(\eta) = \left( \frac{1}{2}(1 + \tanh \rho\eta) \right)^{\frac{i}{2\rho}\omega_{in}} \left( \frac{1}{2}(1 - \tanh \rho\eta) \right)^{\frac{i}{2\rho}\omega_{out}} {}_2F_1 \left( \alpha, \beta, \gamma, \frac{1}{2}(1 + \tanh \rho\eta) \right) \mathbf{V}, \quad (16)$$

$$\Phi_{out}(\eta) = \left( \frac{1}{2}(1 + \tanh \rho\eta) \right)^{\frac{i}{2\rho}\omega_{in}} \left( \frac{1}{2}(1 - \tanh \rho\eta) \right)^{\frac{i}{2\rho}\omega_{out}} {}_2F_1 \left( \alpha, \beta, 1 + \alpha + \beta - \gamma, \frac{1}{2}(1 - \tanh \rho\eta) \right) \mathbf{V}, \quad (17)$$

where

$$\alpha = 1 + \beta, \quad \beta = \frac{i}{2\rho}(\omega_{in} + \omega_{out}), \quad \gamma = 1 + \frac{i\omega_{in}}{\rho}$$

$$\omega_{in}^2 = k^2 + m^2(1 + 2\epsilon), \quad \omega_{out}^2 = k^2 + m^2, \quad (18)$$

and  $\mathbf{V}$  is a constant vector of dimension  $3 \times 1$ .  $\mathcal{X}(\eta)$  and  $\Theta(\eta)$  can be evaluated easily from (15).  ${}_2F_1(a, b, c, d)$ 's are hypergeometric functions. Using these functions properties, the above solutions will be related to each other by well-known Bogoliubov transformation technique as

$$\Phi_{in} = \alpha_k \Phi_{out} + \beta_k \Phi_{out}^*, \quad (19)$$

where,  $\alpha_k$  and  $\beta_k$  are Bogoliubov coefficients. Since  $\mathcal{X}(\eta)$  and  $\Theta(\eta)$  are given by  $\Phi(\eta)$ , one can relate the *in* and *out* modes of the wavefunction by a similar equation as follows

$$\Psi_{in} = \alpha_k \Psi_{out} + \beta_k \Psi_{out}^*. \quad (20)$$

Using the properties of hypergeometric functions [25], we evaluate the Bogoliubov coefficients

$$\alpha_k = \frac{\Gamma \left( 1 + \frac{i\omega_{in}}{\rho} \right) \Gamma \left( -\frac{i\omega_{out}}{\rho} \right)}{\Gamma \left( 1 + \frac{i(\omega_{in} - \omega_{out})}{2\rho} \right) \Gamma \left( \frac{i(\omega_{in} - \omega_{out})}{2\rho} \right)}, \quad (21)$$

$$\beta_k = \frac{\Gamma \left( 1 + \frac{i\omega_{in}}{\rho} \right) \Gamma \left( \frac{i\omega_{out}}{\rho} \right)}{\Gamma \left( 1 + \frac{i(\omega_{in} + \omega_{out})}{2\rho} \right) \Gamma \left( \frac{i(\omega_{in} + \omega_{out})}{2\rho} \right)}. \quad (22)$$

Now, we can work out a relation between the annihilation and creation operators in *in* and *out* regions as follows

$$a_k^{in} = \alpha_k^* a_k^{out} + \beta_k^* b_k^{\dagger out}, \quad (23)$$

where,  $a$  and  $a^\dagger$  ( $b$  and  $b^\dagger$ ) are the annihilation and creation operators of particles (anti-particles) and satisfy the following well-known relations

$$[a_k^{\dagger in(out)}, a_{k'}^{in(out)}] = \delta(k - k'),$$

$$[b_k^{\dagger in(out)}, b_{k'}^{in(out)}] = \delta(k - k'). \quad (24)$$

obtain the following two different solutions, which are analytic in *in* region,  $\Phi_{in}$ , for distant past, and *out* region,  $\Phi_{out}$ , for far future.

The five dimensional representations of Kemmer matrices are given by

$$\beta^0 = \begin{pmatrix} \sigma_x & \mathbf{0}_{2 \times 3} \\ \mathbf{0}_{3 \times 2} & \mathbf{0}_{3 \times 3} \end{pmatrix}, \quad \beta^i = \begin{pmatrix} \mathbf{0}_{2 \times 2} & \rho^i_{2 \times 3} \\ \rho^{i\dagger}_{3 \times 2} & \mathbf{0}_{3 \times 3} \end{pmatrix}, \quad (25)$$

where,  $\sigma_x$  is the Pauli matrix and  $\rho^i_{jk} = -\delta_{1j}\delta_{ik}$ . In this case,  $\tilde{\Psi}(\eta)$  has five  $\tilde{\Psi}_i$  components, where  $i = 1, \dots, 5$ . Therefore, DKP equation reduces to the following equations

$$iCm\tilde{\Psi}_1 = -(\partial_\eta - \frac{\dot{C}}{C})\tilde{\Psi}_2 + ik\tilde{\Psi}_3,$$

$$iCm\tilde{\Psi}_2 = -\partial_\eta\tilde{\Psi}_1,$$

$$iCm\tilde{\Psi}_3 = -ik\tilde{\Psi}_1, \quad (26)$$

and the components  $\tilde{\Psi}_4 = \tilde{\Psi}_5 = 0$ .  $\tilde{\Psi}_1$  satisfies Eq. (14), and  $\tilde{\Psi}_2$  and  $\tilde{\Psi}_3$  satisfy the same Eqs. (15). Therefore, the solutions of the above equations with respect to the *in* and *out* modes are the same as for spin-1 case and they are related to each other by the same Bogoliubov coefficients. This is a significant point achieved in the rest of the paper.

### III. ENTANGLEMENT GENERATION DUE TO EXPANSION

We consider the vacuum state in *in* region as a separable state [14]

$$|0\rangle^{in} = \prod_{k \in \mathbb{R}^+} |0_k\rangle^{in} |0_{-k}\rangle^{in}. \quad (27)$$

For simplicity, we focus on a specific momentum,  $k$ , mode as follows

$$|0\rangle_k^{in} = |0_k\rangle^{in} |0_{-k}\rangle^{in}. \quad (28)$$

In other words, we disregard all other modes of vacuum by tracing out the total density matrix over them. Using

Eq. (23), we write the above bipartite separable state as a linear combination of the excited states in mode  $k$  by an observer in *out* region

$$|0_k\rangle^{in}|0_{-k}\rangle^{in} = \sum_{n=0}^{\infty} A_n |n_k\rangle^{out}|n_{-k}\rangle^{out}. \quad (29)$$

Obviously, the last relation is the Schmidt decomposition of a pure state of a bipartite system. The Schmidt coefficient is obtained by the normalization condition and by applying the annihilation operator on the state

$$a_k^{in}|0\rangle_k^{in} = (\alpha_k^* a_k^{out} - \beta_k^* b_k^{\dagger out}) \sum_{n=0}^{\infty} A_n |n_k\rangle^{out}|n_{-k}\rangle^{out} = 0. \quad (30)$$

Therefore, one finds the following relation for the coefficients

$$A_n = \left(\frac{\beta_k^*}{\alpha_k^*}\right)^n A_0, \quad A_0 = \sqrt{1 - \left|\frac{\beta_k}{\alpha_k}\right|^2}. \quad (31)$$

Thus, a vacuum state in *in* region corresponds to a state with particle excitations in *out* region. The particle creation via the expansion is a well-known concept [26]. In the following, we concentrate on the entanglement generation related to this concept.

We construct the density matrix in *in* region by using Eq. (29) in *out* region

$$\rho_{k,-k} = |0_k\rangle^{in}|0_{-k}\rangle^{in}\langle 0_k|^{in}\langle 0_{-k}|^{in}. \quad (32)$$

Because the Schmidt coefficients in Eq. (29) are non-zero, the *in* vacuum is entangled from the point of view of an observer in *out* region.

We use an appropriate measure of entanglement, namely the von Neumann entropy defined as follows

$$S(\rho_k) = -\text{tr}(\rho_k \log_2(\rho_k)), \quad (33)$$

where,  $\rho_k$  is the reduced density matrix of the particles' subsystem

$$\rho_k = \text{tr}_{-k}(\rho_{k,-k}). \quad (34)$$

The von Neumann entropy for spin-1 modes is given by

$$S = - \sum_{n=0}^{\infty} |A_n|^2 \log_2 |A_n|^2 = \log_2 \frac{x^{\frac{x}{x-1}}}{1-x}, \quad (35)$$

where,  $x = \left|\frac{\beta_k}{\alpha_k}\right|^2$  and  $\alpha_k$  and  $\beta_k$  are given by Eqs. (21) and (22), respectively.

Fig. (1) shows the contour plot of the von Neumann entropy with respect to the mass and the momentum of spin-1 particles for the specified expanding universe (fixed values of total volume and rapidity of expansion). The generated entanglement is a decreasing function with respect to the momentum. Also, there is no entanglement

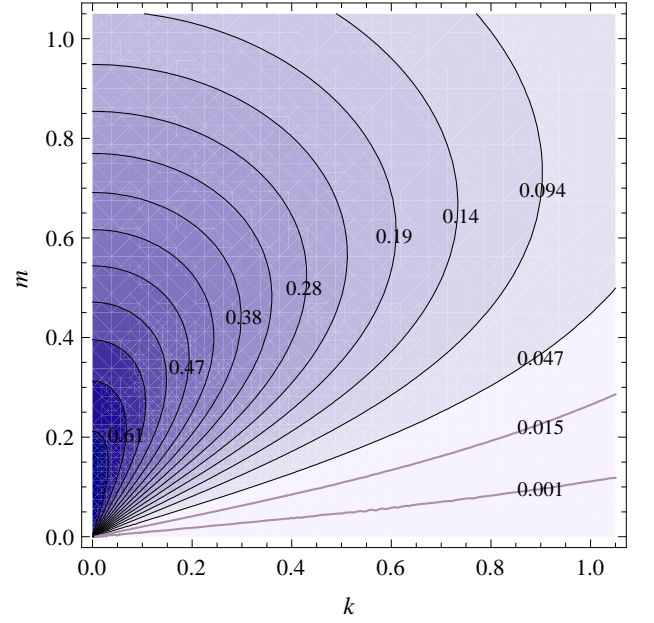


FIG. 1: (Color online) Contour plot of the von Neumann entropy for spin-1 modes with respect to the mass and momentum for the specified expanding universe  $\epsilon = 2$  and  $\rho = 2$ .

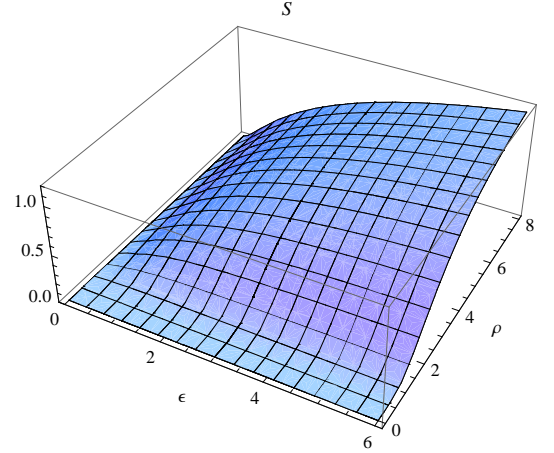


FIG. 2: (Color online) The von Neumann entropy for spin-1 modes with respect to the total volume and rapidity of expansion for  $m = 1$  and  $k = 0.1$ .

between massless spin-1 particles. For each value of momentum, there exists a specific value of mass,  $m_{\max}$ , at which the entanglement is maximum.  $m_{\max}$  is an increasing function with respect to the momentum.

It is obvious from Fig. (2) that for the specified mass and momentum of spin-1 particles, the generated entanglement is an increasing function with respect to the total volume,  $\epsilon$ , and the rapidity of expansion,  $\rho$ . There is no entanglement generation for  $\epsilon = 0$  and  $\rho = 0$ , which correspond to a flat Mikowskian space-time.

In addition, one can observe from Fig. (3) that there is a certain value of mass,  $m_{\max}$ , wherein the generated

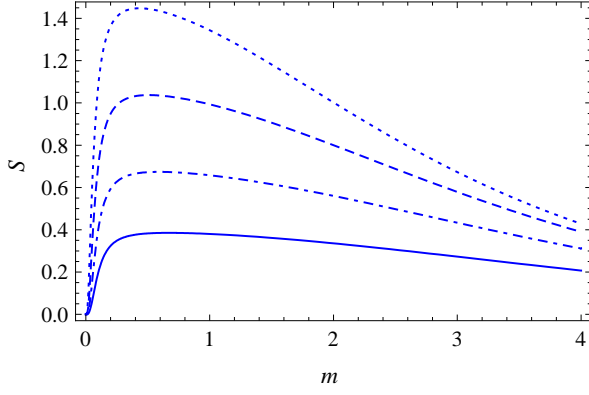


FIG. 3: (Color online) The von Neumann entropy,  $S$ , for spin-1 modes with respect to mass,  $m$ , for fixed values of momentum,  $k = 0.1$ , and rapidity,  $\rho = 10$ . The different values of total volume are  $\epsilon = 1$  (solid),  $\epsilon = 2$  (dash dotted),  $\epsilon = 4$  (dashed), and  $\epsilon = 8$  (dotted) curves.

entanglement is maximum. Naturally, the entanglement generation is larger for an expanding universe with a larger total volume expansion. In a similar behavior,  $m_{\text{max}}$  tends to a larger value for larger total volume expansion.

The above arguments and results were represented for spin-1 particles. Since the von Neumann entropy is a function with respect to the Bogoliubov coefficients and we argue in section II that these coefficients are the same for spin-1 and spin-0 particles, the general behavior of entanglement generation for spin-0 particles will be the same as spin-1 particles.

#### IV. SPIN-3/2 PARTICLES AND ENTANGLEMENT GENERATION

The Rarita-Schwinger equation characterizes the dynamics of massive spin-3/2 particles in flat Minkowski space-time [27]. In supergravity models, the superpartner of graviton field is described by spin-3/2 particles. The complicated form of the Rarita-Schwinger equation makes it inexplicable to obtain explicit results even in a simple background. It has been shown that when one considers helicity  $\pm 3/2$  states propagating in arbitrary homogeneous and isotropic scalar or gravitational backgrounds, the equations can be reduced to a Dirac-like equation in flat Minkowski space-time [28, 29] as follows

$$(i \not{\partial} - m)\psi_\mu = 0, \quad (36)$$

with two constraints

$$\begin{aligned} \gamma^\mu \psi_\mu &= 0, \\ \partial^\mu \psi_\mu &= 0. \end{aligned} \quad (37)$$

By replacing ordinary derivatives by covariant ones, we obtain the equation for FRW metrics as follows [29]

$$\begin{aligned} (i \not{D} - m)\psi_\mu &= 0, \\ \gamma^\mu \psi_\mu &= 0, \\ D^\mu \psi_\mu &= 0. \end{aligned} \quad (38)$$

Where,  $D_\rho \psi_\sigma = (\partial_\rho + \frac{i}{2} \Omega_\rho^{ab} \Sigma_{ab})\psi_\sigma$  with  $\Omega_\rho^{ab}$  the spin connection coefficients and  $\Sigma_{ab} = \frac{i}{4}[\gamma_a, \gamma_b]$  and  $[D_\mu, D_\nu] = -\frac{i}{2} R_{\mu\rho}^{ab} \Sigma_{ab}$ . According to Eq. (8) for FRW expanding universe, we obtain the spin-connections, Riemann tensor and finally the following equation

$$\left( iC^{-1} \gamma^\mu \partial_\mu - m + i \frac{3}{2} \frac{\dot{C}}{C^2} \gamma^0 \right) \psi_\mu = 0. \quad (39)$$

By rewriting  $\psi_\mu$  as a multiplication of spatial part,  $\exp(i\vec{k} \cdot \vec{x})$  and time dependent function,  $\kappa(\eta)$ , also an appropriate function of  $C(\eta)$ , we obtain the following equation for time dependent part [29]

$$\left( \frac{d^2}{d\eta^2} + k^2 - im\dot{C} + m^2 C^2 \right) \kappa(\eta) = 0. \quad (40)$$

Comparison between Eq. (40) and the same evaluation for spin-1/2 particles in curved space-time [30] shows that two distant past and far future asymptotic solutions are related by the same Bogoliubov coefficients. It is straightforward to show that the von Neumann entropy is given by

$$S = - \sum_{n=0}^1 |A_n|^2 \log_2 |A_n|^2 = \log_2 \frac{1+x}{x \frac{x}{1+x}}, \quad (41)$$

where,  $x = |\frac{\beta_k}{\alpha_k}|^2$ ,  $\alpha_k$  and  $\beta_k$  are Bogoliubov coefficients. Since von Neumann entropy is directly related to these coefficients, we can argue that the generated entanglement of spin-3/2 particles due to the expanding universe will have the same properties of spin-1/2 case.

#### V. CONCLUSION

DKP equation is employed for considering the spin-1 particles in the FRW expanding universe. It is showed that there is two asymptotic distant past and far future times and one can connect these solutions to each other by the well-known Bogoliubov technique. The separable vacuum state of distant past time can be appeared as an entangled particle-antiparticle state in far future time. In order to measure the entanglement of the generated pure state, the von Neumann entropy is used. The general behavior of the generated entanglement is summarized with respect to the characteristics of an expanding universe, the mass and the momentum of any modes.

It is also shown that the general behavior of the generated entanglement of spin-0 particles is the same as spin-1 particles because of the same Bogoliubov coefficients which relate the asymptotic solutions to each other.

We considered spin-3/2 particles in FRW space-time using the Rarita-Schwinger equation. Comparing the equation which describes the time dependent part of the field with the spin-1/2, one shows that Bogoliubov coefficients for both spin-3/2 and spin-1/2 will be the same. Therefore, the general behavior of the generated entanglement will also be the same.

Investigation of the generated entanglement in an ex-

panding universe for spin-0 and spin-1/2 particles shows some deep differences between these modes. Our consideration in this paper proposes that these differences are independent from the absolute value of the spin of the particles. We argue that the absolute value of a spin does not play any role in entanglement generation and the differences are due to the bosonic or fermionic properties.

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